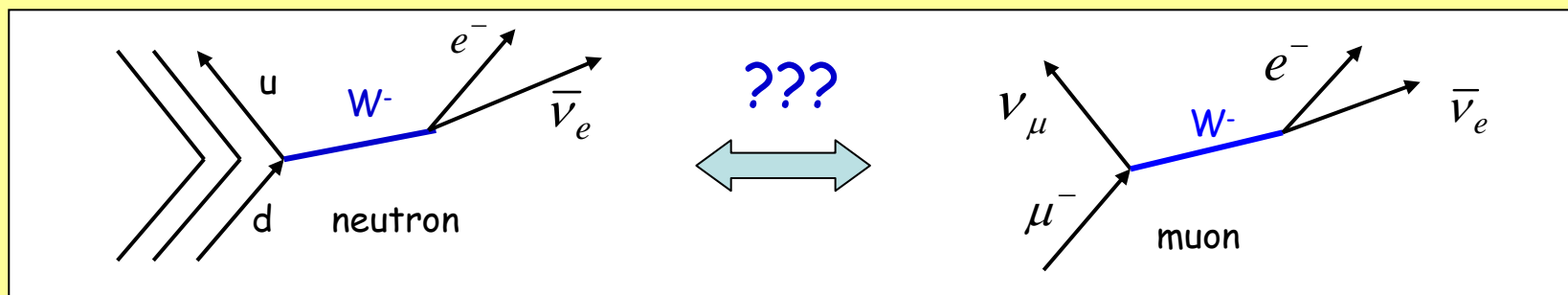


We have, so far two coupling constants for (nuclear) beta decay: G_V ($S=0$ decays) and G_A ($S=1$). These set the overall scale of the interaction, with G_V determined from the transition rates for "superallowed" $0^+ \rightarrow 0^+$ nuclear decays, and G_A from Gamow-Teller decays ($0^+ \rightarrow 1^+$ and vice versa).

Other related processes:

1. **muon decay:** $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$ or $\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$

- a "purely leptonic" weak decay -- no quarks before or after!
- no change of electric charge; must be "mediated" by the neutral Z^0 boson
- no "Fermi function" needed, since no Coulomb effects in the final state.
- analogous to neutron decay, so we can try the same formalism, assuming weak interactions for quarks and leptons are the same



Muon lifetime implications:

2

measured lifetime: $\tau = 2.19703 \pm 0.00004 \text{ } \mu\text{s}$

theoretical prediction:

$$\tau = \frac{192 \pi^3 \hbar^7}{G^2 m_\mu^5 c^4}$$

$$\mu^\pm \rightarrow e^\pm + \nu_e / \bar{\nu}_e + \bar{\nu}_\mu / \nu_\mu$$

(our prediction, integrated over phase space for the two neutrino types!)

Muon decay gives a weak **coupling constant G** that is about **2.5% larger** than in nuclear beta decays....

or alternatively, the coupling constant for the $d \rightarrow u$ quark weak transition is about 2.5% smaller than that for the $\mu \rightarrow e$ lepton weak transition.

2. Pion decay:

$$\pi^- \rightarrow \pi^0 + e^- + \bar{\nu}_e$$

Another $d \rightarrow u$ quark transition; rate is consistent with the **same** coupling constants as nuclear beta decay

3. K meson decay

$$K^- \rightarrow \pi^0 + e^- + \bar{\nu}_e \quad (\bar{u}s \rightarrow \bar{u}u + e^- + \bar{\nu}_e)$$

This is an **$s \rightarrow u$** quark transition; rate is **much smaller** than the equivalent $d \rightarrow u$ rate; **coupling constants are reduced to about 20% of nuclear beta decay values**

Weak interactions of quarks:

- There are hundreds of examples of weak decays in nuclear and particle physics.
- **Purely leptonic rates** are all consistent with a **single weak coupling constant G**
- **Hadronic rates**, involving quark transitions, occur at a comparable scale but with consistent differences that depend on the type of quarks involved.
- A simple pattern emerges if we assume that **the quarks that participate in weak interactions are linear combinations of the strong interaction eigenstates**, represented by a **unitary matrix** called the CKM (Cabbibo-Kobayashi-Maskawa) matrix:

$$\begin{bmatrix} d' \\ s' \\ b' \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \times \begin{bmatrix} d \\ s \\ b \end{bmatrix}$$

weak eigenstates
strong eigenstates

Unitary matrix, like a rotation matrix - preserves "length"

$$d' = V_{ud} d + V_{us} s + V_{ub} b, \text{ etc...}$$

- Instead of a $d \rightarrow u$ transition in neutron beta decay, **only the contribution from the weak eigenstate d' plays a role**, and the weak coupling constant is effectively reduced by a factor $V_{ud} = 0.974$.
- Similarly, instead of an $s \rightarrow u$ transition in kaon decay, we have an $s' \rightarrow u$ transition, effectively reducing the weak coupling constant by a factor $V_{us} = 0.220$.
- Studies of a large number of particle decays and beta transitions have effectively "mapped out" the CKM matrix as follows: (Particle Data Group, 2004)

$$\left| \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \right| = \begin{bmatrix} 0.974 & 0.220 & 0.004 \\ 0.224 & 0.996 & 0.041 \\ 0.009 & 0.041 & 0.999 \end{bmatrix}$$

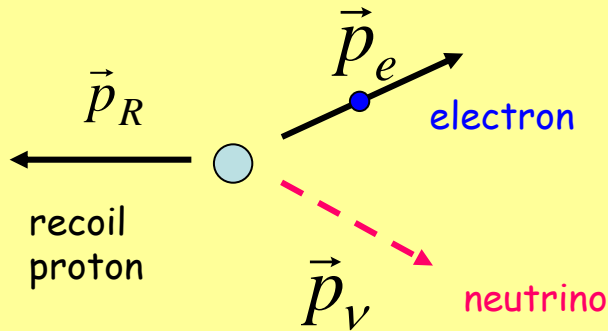
0.9739 to 0.9751	0.221 to 0.227	0.0029 to 0.0045
0.221 to 0.227	0.9730 to 0.9744	0.039 to 0.044
0.0048 to 0.014	0.037 to 0.043	0.9990 to 0.9992

2σ limits

- **Diagonal terms dominate the CKM matrix**
- All "large" terms are real; small imaginary component in lower right 2×2 submatrix allows for time reversal, or alternatively "CP violation" -- a hot research topic!

Finally, what can we learn from neutron decay?

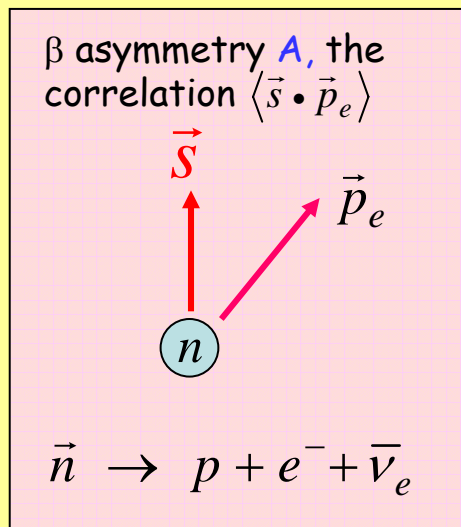
5



measuring **spin-momentum correlations** for the decay of **polarized** neutrons yields additional information (**neutron spin: \vec{s}**)
 → correlation coefficients: **a , A , B** :

$$A = -2 \frac{-G_A G_V + G_A^2}{G_V^2 + 3G_A^2}, \quad \tau = \frac{\text{constant}}{G_V^2 + 3G_A^2}$$

$$\lambda_{if} \propto p_e E_e (Q - E_e)^2 \left[1 + a \frac{\vec{p}_e \cdot \vec{p}_\nu}{E_e E_\nu} + \langle \vec{s} \rangle \cdot \left(A \frac{\vec{p}_e}{E_e} + B \frac{\vec{p}_\nu}{E_\nu} \right) \right] dE_e d\Omega_e d\Omega_\nu$$



"**little a** " and " **B** " are hard to measure because one cannot determine the neutrino momentum directly.

The **best additional measurement** is the "**big A** " coefficient, which gives an independent constraint from the neutron lifetime, but **one has to control and measure the neutron spin direction and measure the electron momentum / energy very precisely...**

new experiment with ultra cold neutrons:

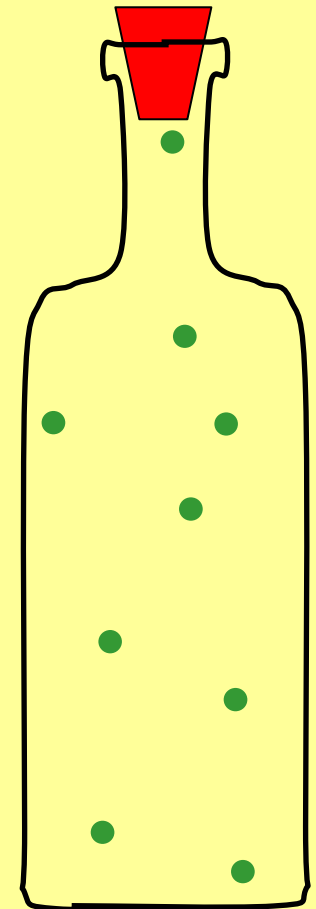
<http://www.krl.caltech.edu/ucn/>



What are Ultra Cold Neutrons (UCN) ?

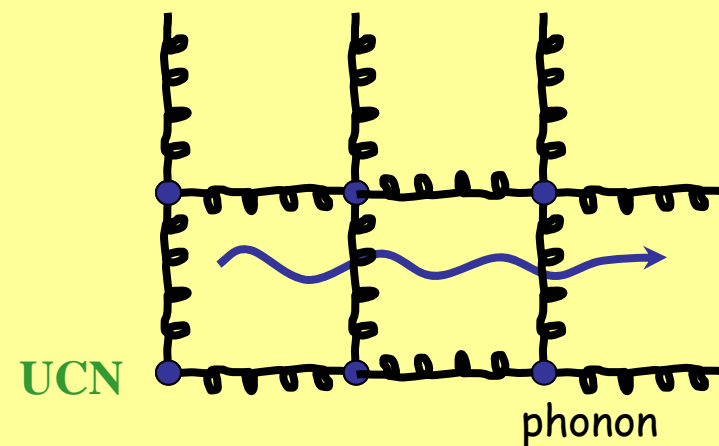
6

- UCN are neutrons that are moving so slowly that they are totally reflected from a variety of materials.
- They can be confined in material bottles for long periods of time.
- Typical parameters:
 - velocity < 8 m/s
 - temperature < 4 mK
 - kinetic energy < 300 neV
- Interactions:
 - gravity: $V=mgh$
 - **weak interaction** (allows UCN to decay)
 - magnetic fields: $V=-\mu \cdot B$
(100 % polarization by passing through a magnet !)
 - strong interaction

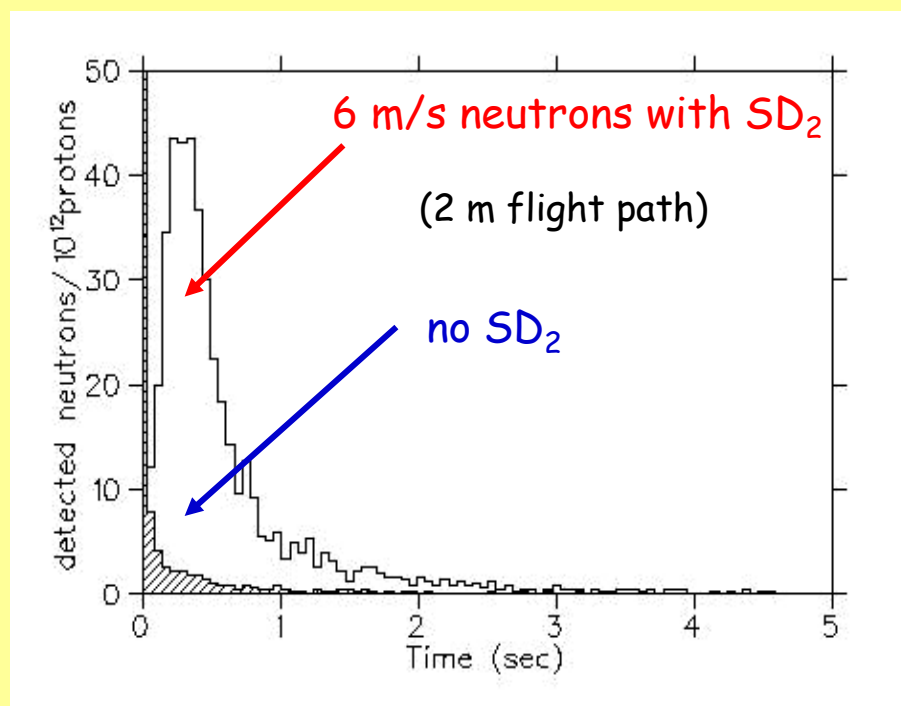
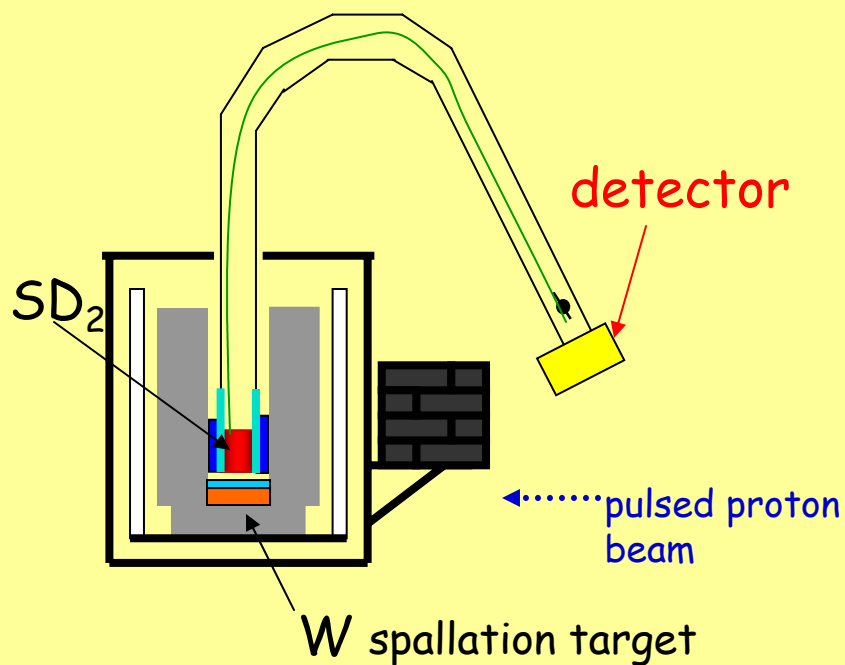


slides courtesy Prof. J. Martin, U. Wpg.

●
Cold n



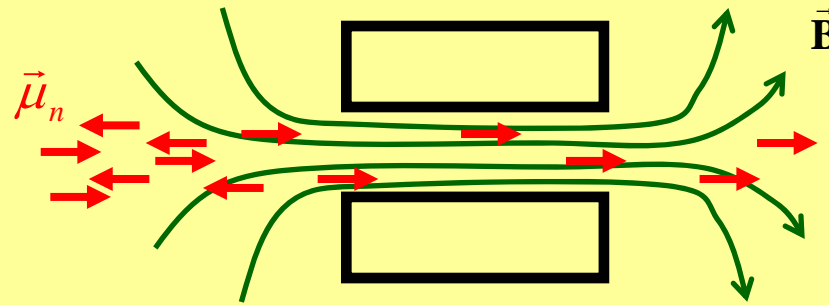
First UCN generation at Los Alamos:



UCNA Advantages: Polarization and Background

8

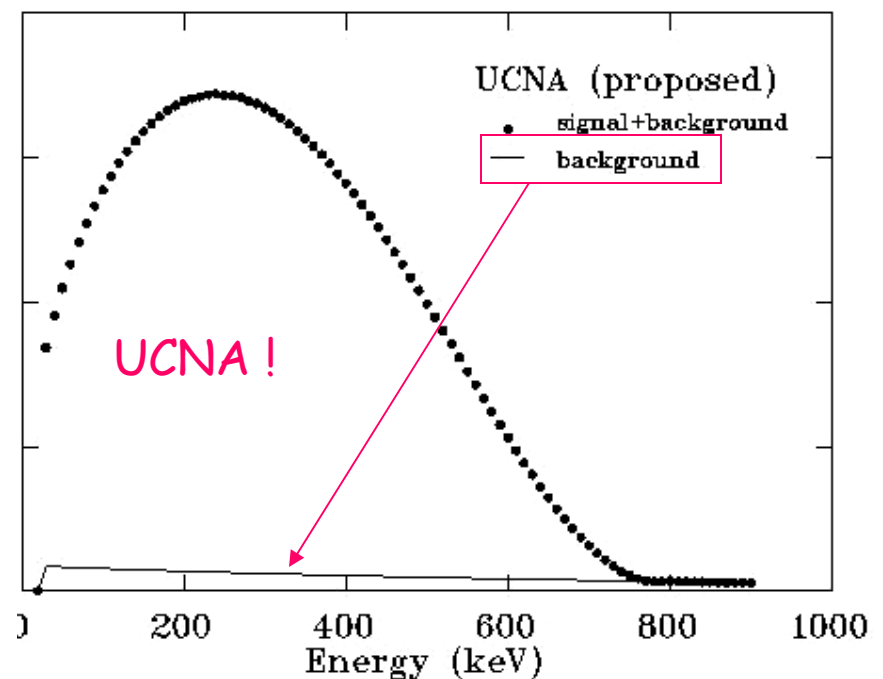
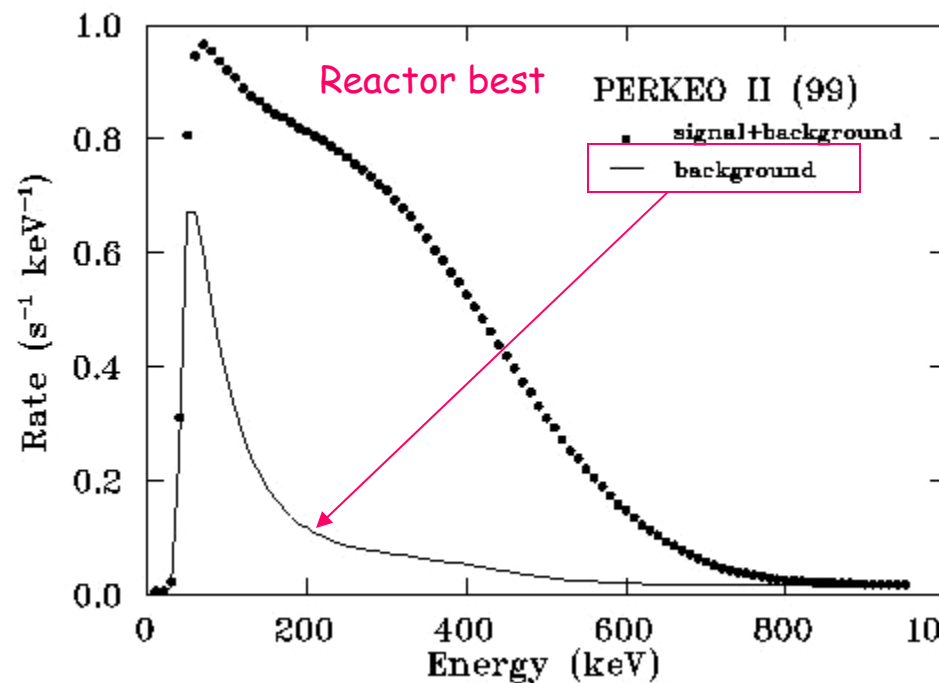
Ultra cold neutrons with the wrong spin direction can't make it through a large magnetic field!

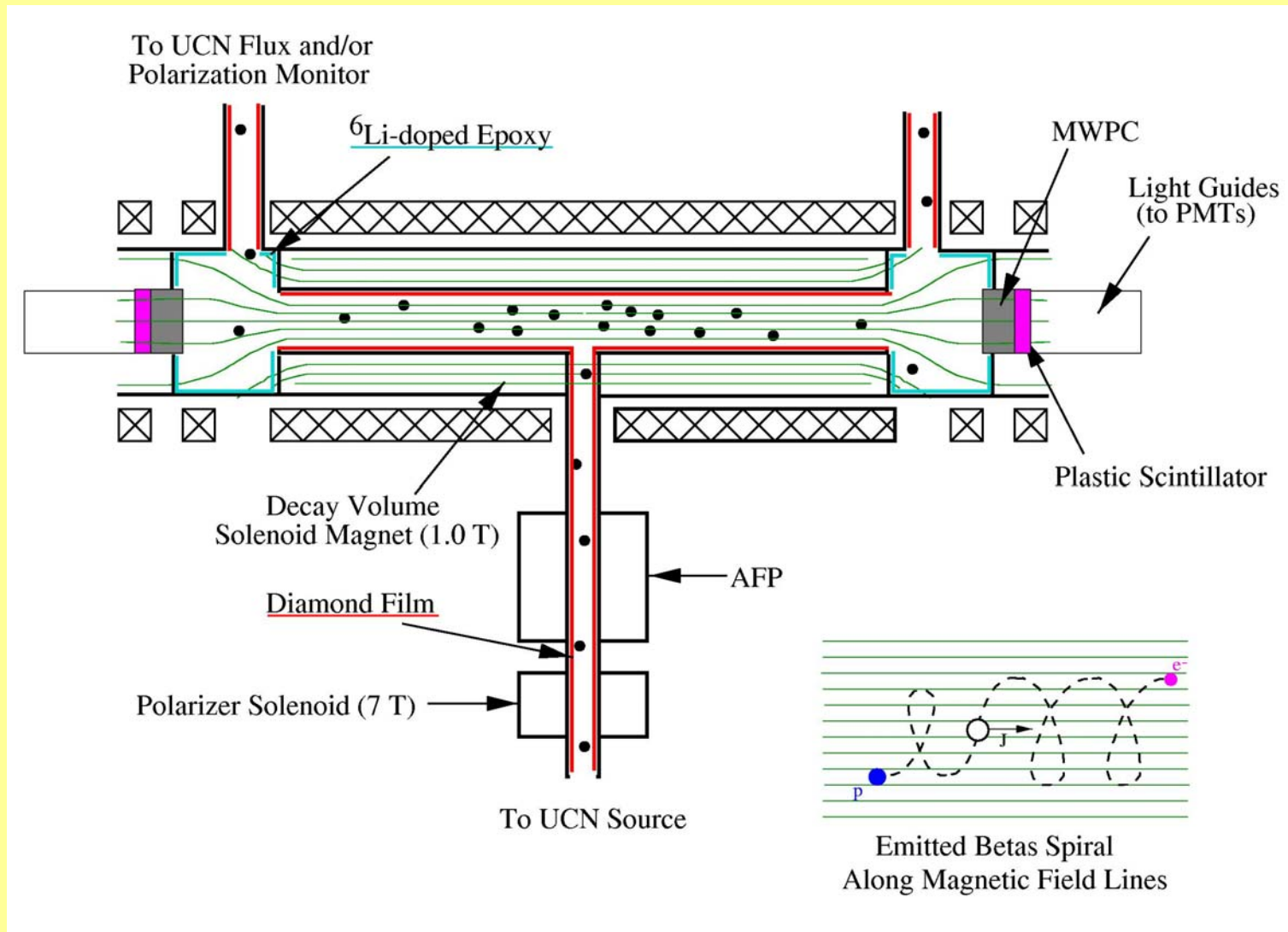


$B > 6 \text{ T}$
 $\rightarrow P = 100\%$

limitation - magnetic scattering from walls, etc.

Background reduction via pulsed source:





Issues: electron backscattering from detector surface (similar issue in lifetime expt.)
neutron depolarization by scattering from the walls ($\sim 0.1\%$)

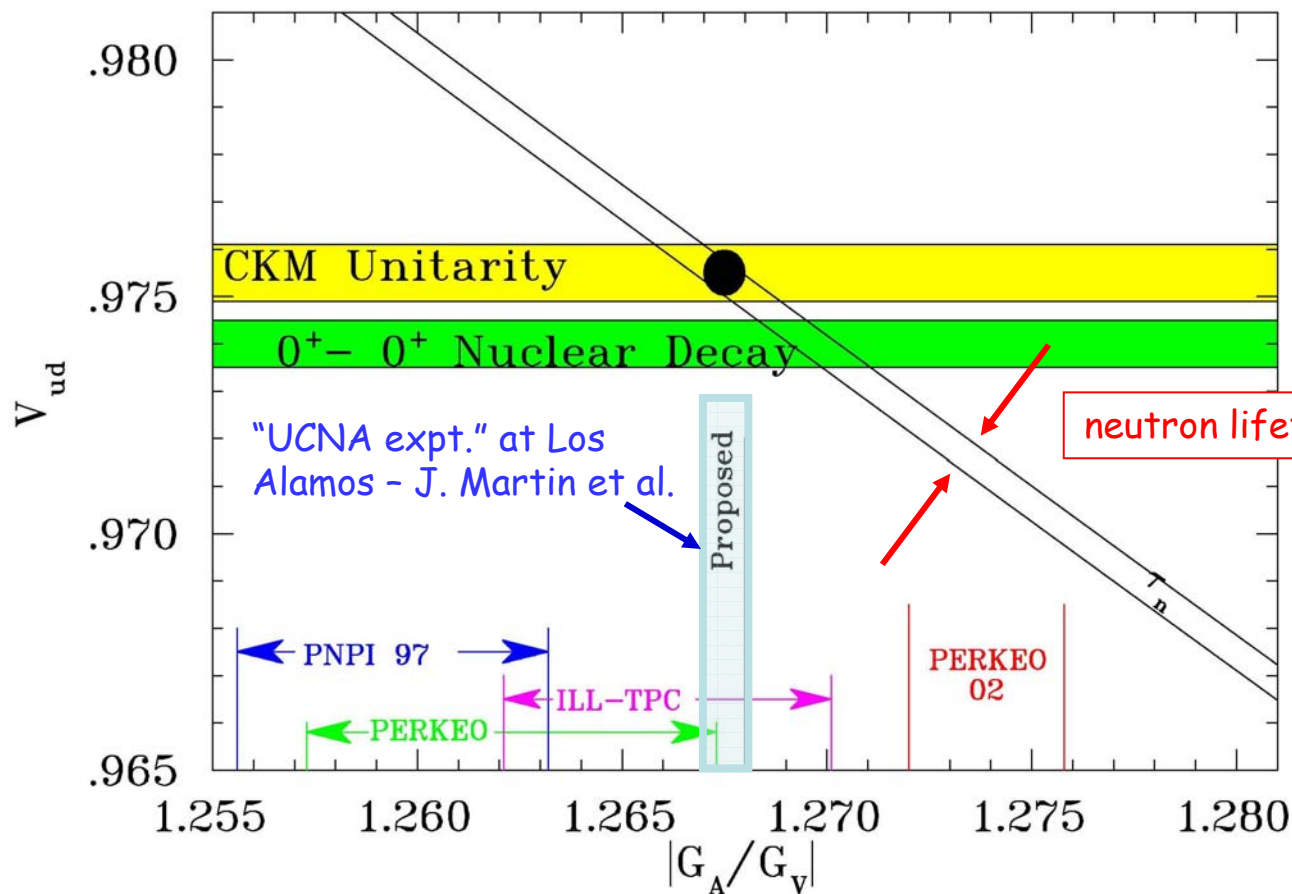
CKM Unitarity test: are there more than three generations of quarks?

10

$$V_{ij}^{-1} = V_{ij}^* \rightarrow V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 1 \quad ? \quad (\text{the best-tested row of } V_{ij})$$

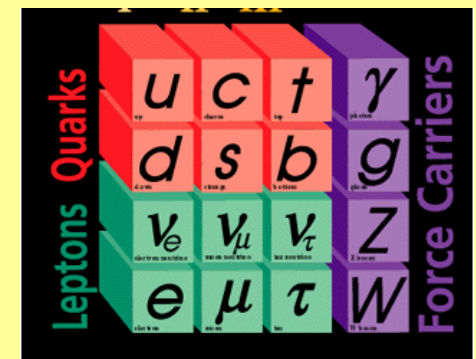
world data, 2004: $V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 0.9967 \pm 0.0014$

**2 σ discrepancy:
contentious issue...**



an additional generation of quarks is not entirely ruled out by the CKM unitarity test at the present time!

neutron lifetime (diagonal band)



horizontal scale - measurements of "A" in neutron decay

Nuclear Science

Expansion of the Universe

After the Big Bang, the universe expanded and cooled. It shows 10^5 seconds the universe contained a soup of quarks, gluons, electrons, and neutrinos. When the temperature of the Universe, T_{universe} , cooled to about 10^9 K, the soup solidified into protons, neutrons, and electrons. As time progressed, most of the protons and neutrons formed deuterium, helium, and lithium nuclei. Lighter elements combined with protons and these formed nuclei in three sequential years. Due to gravity, clouds of atoms contracted over time, where hydrogen and helium formed from very massive chemical elements. Expanding stars (supernovae) from the most massive elements and shaped these into space. Our earth was formed from supernova debris.



Radioactivity

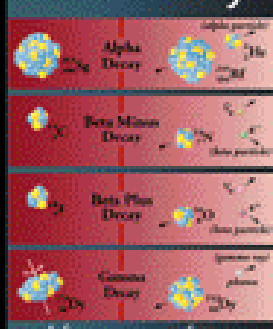


Chart of the Nuclides

The Chart of the Nuclides presents a graphic view of known nuclei with atomic number Z , and neutron number N . Each nuclide is represented by a box colored according to its predominant decay mode. Stable nuclides (Z or $N = 2, 8, 18, 28, 50, 82$ and 126) are indicated by a white box in the chart. The chart is color-coded to show regions of greater nuclear binding energy.

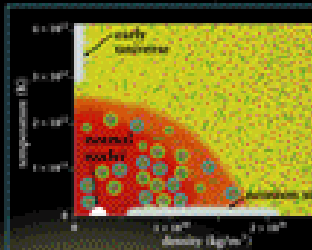


www.CPEPweb.org

Nuclear Science is the study of the structure, properties, and interactions of the atomic nuclei. Nuclear scientists calculate and measure the masses, shapes, sizes, and lifetimes of nuclei on a scale from billions to fractions. They ask questions, such as "Why do nuclei stay together?" "What combinations of protons and neutrons are possible?" "What happens when nuclei are compressed or rapidly moved?" "What is the origin of the nuclei found on Earth?"

Legend

- quarks (u, d)
- gluons (g)
- electrons (e^-)
- positrons (e^+)
- neutrons (n)
- protons (p)
- photons (γ)
- neutrinos (ν)



Phases of Nuclear Matter

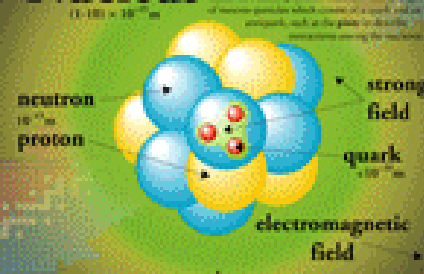
Nuclei exist in various phases. When subjected to extreme conditions, individual protons and neutrons may separate from the nuclear fluid. At sufficiently high temperatures or densities, or at low densities and high temperatures, individual nucleons may exist as free, unbound particles, moving independently. Current data suggests that quark-gluon plasma exists at high temperatures and low densities. It is believed that quark-gluon plasma exists at high temperatures and low densities.

Unstable Nuclei

Nuclei exist in a state where they are not stable. The Nucleus Science program studies nuclei that are unstable. These nuclei decay, changing their composition of protons and neutrons. In the process, they release energy. Some 2000 different nuclei have been identified. Nuclei decay products that are not stable may further decay into stable nuclei.



The Nucleus



In an atom, electrons range around the nucleus at distances typically up to 10,000 times the nuclear diameter. If the electrons were removed, the nucleus would collapse to a small size.

Nuclear Energy



In the early stages of nuclear reactions, the nuclei are very hot and move rapidly. The nuclei are held together by a strong field. The nuclei are also surrounded by an electromagnetic field. The nuclei are held together by a strong field. The nuclei are also surrounded by an electromagnetic field.

Applications



Radioactive Dating
Nuclei existing within rocks can be used to date the rocks. The nuclei are held together by a strong field. The nuclei are also surrounded by an electromagnetic field.



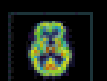
Space Exploration
Nuclei existing within rocks can be used to date the rocks. The nuclei are held together by a strong field. The nuclei are also surrounded by an electromagnetic field.



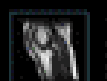
Nuclear Reactors
Nuclei existing within rocks can be used to date the rocks. The nuclei are held together by a strong field. The nuclei are also surrounded by an electromagnetic field.



Smoke Detectors
Nuclei existing within rocks can be used to date the rocks. The nuclei are held together by a strong field. The nuclei are also surrounded by an electromagnetic field.



Nuclear Medicine
Nuclei existing within rocks can be used to date the rocks. The nuclei are held together by a strong field. The nuclei are also surrounded by an electromagnetic field.



Magnetic Resonance Imaging
Nuclei existing within rocks can be used to date the rocks. The nuclei are held together by a strong field. The nuclei are also surrounded by an electromagnetic field.

Basic properties:

$$d \equiv {}^2_1\text{H}$$

mass: $mc^2 = 1876.124 \text{ MeV}$

binding energy: $B \equiv \sum_i m_i - M = m_p + m_n - m_d = 2.2245731 \text{ MeV}$
 (measured via γ -ray energy in $n + p \rightarrow d + \gamma$)

RMS radius: $1.963 \pm 0.004 \text{ fm}$ (from electron scattering)

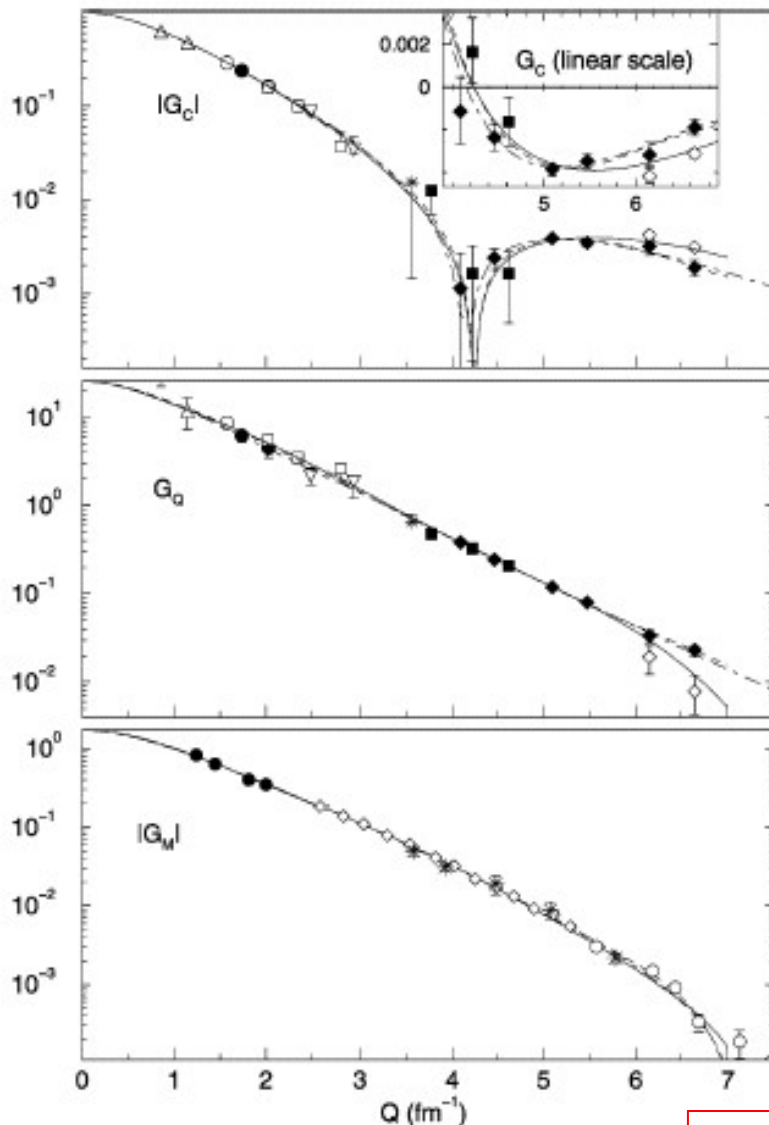
quantum numbers: $J^\pi, I = 1^+, 0$ (lectures 13, 14)

magnetic moment: $\mu = +0.8573 \mu_N$

electric quadrupole moment: $Q = +0.002859 \pm 0.00030 \text{ bn}$
 (\rightarrow the deuteron is not spherical!)

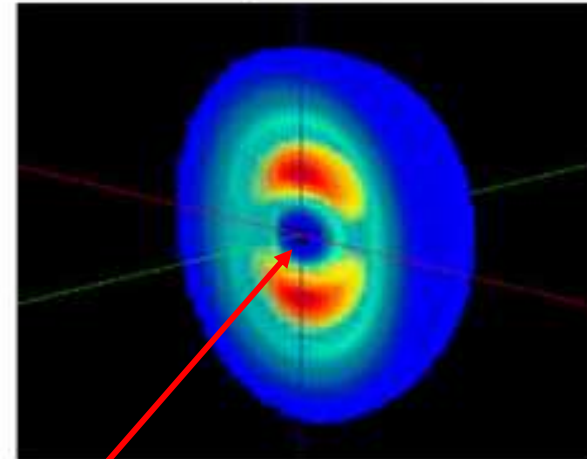
Important because:

- deuterium is the lightest nucleus and the only bound N-N state
- testing ground for state-of-the art models of the N-N interaction.



Because the deuteron has spin 1, there are 3 form factors to describe elastic scattering: the "charge" (G_c), "electric quadrupole" (G_q) and "magnetic" (G_M) form factors. (JLab data)

Combined Data \Rightarrow
Intrinsic Shape of the Deuteron



Interesting feature: strong attractive np force, but a void in the center - the deuteron is hollow! ... **Why?**

$$\vec{S} \equiv \vec{S}_n + \vec{S}_p$$

$$\vec{J} = \vec{S} + \vec{L} = \vec{1}$$

$$\pi = (+)(+)(-1)^L = + \Rightarrow L = 0, 2, 4 \dots$$

- Of the possible quantum numbers, $L = 0$ has the lowest energy, so we expect the ground state to be $L = 0, S = 1$ (the deuteron has no excited states!)
- The nonzero electric quadrupole moment suggests an admixture of $L = 2$ (more later!)

introduce **Spectroscopic Notation:** $^{2S+1}L_J$

with naming convention: $L = 0$ is an S-state, $L = 1$ is a P-state, $L = 2$: D-state, etc...

→ the deuteron configuration is primarily 3S_1

$$\vec{I} = \frac{\vec{1}}{2} + \frac{\vec{1}}{2} \Rightarrow I = 0, 1$$

The total wavefunction for two identical Fermions has to be **antisymmetric** w.r.to particle exchange:

$$\Psi_{total} = \psi_{space} \times \phi_{spin} \times \chi_{isospin}$$

Central force problem:

$$\psi_{space}(r, \theta, \phi) = f(r) Y_{LM}(\theta, \phi)$$

with **symmetry** $(-1)^L$ given by the spherical harmonic functions

Spin and Isospin configurations:

$S = 0$ and $I = 0$ are antisymmetric ;

$S = 1$ and $I = 1$ are symmetric

3S_1 state can only be $I = 0$!

Magnetic Moment: $\mu_d = 0.857 \mu_N$

In general, the magnetic moment is a quantum-mechanical vector; it must be aligned along the "natural symmetry axis" of the system, given by the total angular momentum:

$$\vec{\mu} \sim \vec{J}$$

But we don't know the direction of \vec{J} , only its "length" and z-projection as expectation values:

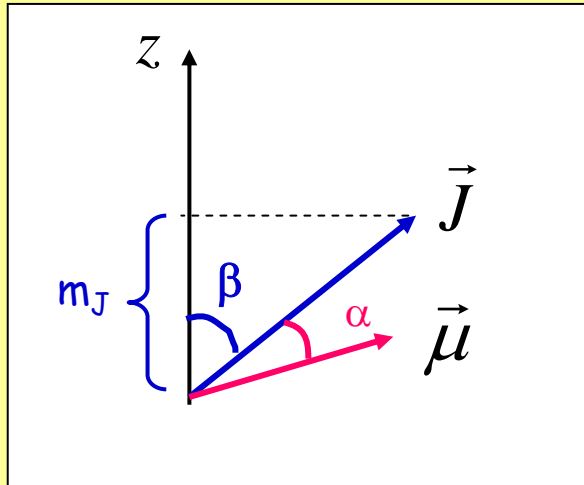
$$\langle J^2 \rangle = J(J+1); \quad \langle J_z \rangle = m_J = (-J \dots + J)$$

in a magnetic field, the energy depends on m_J via $\Delta E = -\langle \vec{\mu} \cdot \vec{B} \rangle \equiv -g_J m_J B \mu_N$

Strategy: we will define the magnetic moment by its maximal projection on the z-axis, defined by the direction of the magnetic field, with $m_J = J$

$$\mu \equiv \langle \vec{\mu} \cdot \hat{z} \rangle \Big|_{m_J=J} = g_J J \mu_N$$

Use expectation values of operators to calculate the result....



$$\mu \equiv \left\langle \vec{\mu} \cdot \hat{z} \right\rangle \Big|_{m_J=J} = g_J J \mu_N$$

Subtle point: we have to make two successive projections to evaluate the magnetic moment according to our definition, and the spin and orbital contributions enter with different weights.

1. Project onto the direction of J :

$$\vec{\mu} \cdot \hat{J} = \mu \cos \alpha = \frac{\vec{\mu} \cdot \vec{J}}{\sqrt{J(J+1)}}$$

2. Project onto the z-axis with $m_J = J$:

$$\cos \beta = \frac{m_J}{|J|} = \frac{J}{\sqrt{J(J+1)}}$$

$$\mu \equiv g_J J \mu_N = \mu \cos \alpha \cos \beta = \left\langle \vec{\mu} \cdot \vec{J} \right\rangle \frac{1}{(J+1)}$$

Next, we need to figure out the operator for $\vec{\mu}$

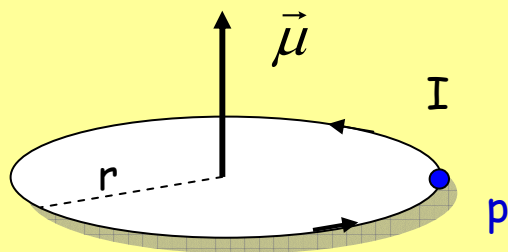
spin and orbital contributions:

We already know the intrinsic magnetic moments of the proton and neutron, so these must correspond to the spin contributions to the magnetic moment operator:

$$\mu_p = +2.79 \mu_N = g_{s,p} S \mu_N \Rightarrow g_{s,p} = +5.58$$

$$\mu_n = -1.91 \mu_N = g_{s,n} S \mu_N \Rightarrow g_{s,n} = -3.83$$

For the orbital part, there is a contribution from the proton only, corresponding to a circulating current loop (semiclassical sketch, but the result is correct)



$$\vec{\mu} = I \pi r^2 \hat{\ell} = g_\ell \vec{\ell} \mu_N, \quad g_\ell = 1$$

For the deuteron, we want to use the magnetic moment operator:

$$\vec{\mu} = \left(g_{s,p} \vec{S}_p + g_{s,n} \vec{S}_n + \vec{L}_p \right) \mu_N$$

1. $\vec{L}_p = \frac{1}{2} \vec{L}$ because $m_n \cong m_p$, and L is the total orbital angular momentum !
2. $L = 0$ in the "S-state" (3S_1) but we will consider also a contribution from the "D-state" (3D_1) as an exercise
3. The proton and neutron couple to $S = 1$, and the deuteron has $J = 1$

$$\mu = \frac{1}{2} \langle \vec{\mu} \cdot \vec{J} \rangle = \frac{1}{2} \left\langle \left(g_{s,p} \vec{S}_p + g_{s,n} \vec{S}_n + \frac{1}{2} \vec{L} \right) \cdot \vec{J} \right\rangle \mu_N$$

Trick: use $\vec{S}_n + \vec{S}_p = \vec{S}$ and write the **operator** as:

$$\vec{\mu} = \left(\frac{1}{2} (g_{s,p} + g_{s,n}) \vec{S} + \frac{1}{2} (g_{s,p} - g_{s,n}) (\vec{S}_p - \vec{S}_n) + \frac{1}{2} \vec{L} \right) \mu_N$$

But the proton and neutron spins are aligned, and $\langle \vec{S}_p \cdot \vec{J} \rangle = \langle \vec{S}_n \cdot \vec{J} \rangle$
so the second term has to give zero!

So, effectively we can write for the deuteron:

$$\mu = \frac{1}{4} \left\langle \left((g_{s,p} + g_{s,n}) \vec{S} + \vec{L} \right) \cdot \vec{J} \right\rangle \mu_N$$

Trick for expectation values:

$$\vec{J} = \vec{L} + \vec{S}; \quad \langle J^2 \rangle = J(J+1)$$

$$\langle J^2 \rangle = \langle (\vec{L} + \vec{S}) \cdot (\vec{L} + \vec{S}) \rangle = \langle L^2 + S^2 + 2\vec{L} \cdot \vec{S} \rangle$$

$$\langle \vec{S} \cdot \vec{J} \rangle = \langle \vec{S} \cdot \vec{L} + \vec{S} \cdot \vec{S} \rangle = \frac{1}{2} \langle J^2 - L^2 - S^2 \rangle + \langle S^2 \rangle = \frac{1}{2} \langle J^2 - L^2 + S^2 \rangle$$

$$\langle \vec{L} \cdot \vec{J} \rangle = \langle \vec{L} \cdot \vec{L} + \vec{L} \cdot \vec{S} \rangle = \frac{1}{2} \langle J^2 + L^2 - S^2 \rangle$$



$$\mu(^3S_1) = \frac{1}{2} (g_{s,p} + g_{s,n}) \mu_N = \mu_p + \mu_n = 0.880 \mu_N$$

$$\mu(^3D_1) = \frac{1}{4} \left(3 - (g_{s,p} + g_{s,n}) \right) \mu_N = 0.310 \mu_N$$

Comparison to experiment:

$$\mu_d = 0.857 \mu_N$$

This is intermediate between the S-state and D-state values:

$$\mu(^3S_1) = 0.880 \mu_N$$

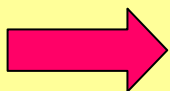
$$\mu(^3D_1) = 0.310 \mu_N$$

Suppose the wave function of the deuteron is a linear combination of S and D states:

$$|\psi_d\rangle = a |^3S_1\rangle + b |^3D_1\rangle \quad \text{with} \quad a^2 + b^2 = 1$$

Then we can adjust the coefficients to explain the magnetic moment:

$$\mu_d = (1-b^2) \mu(^3S_1) + b^2 \mu(^3D_1)$$



$b^2 = 0.04$, or a 4% D-state admixture accounts for the magnetic moment !